Lecture 1

Tuesday, April 26, 2022 8:18 PM

* Prayer
* Spiritual thought

- Outline of the course:

  Course website & Learning Suite

  Written HW → 30% (bonus points available) → Mathematica access

  Online HW → 10% → WebAssign access → Textbook

  Midterm → 30% → May 19-21

  Final → 30% (non-comprehensive) → June 15 @ 5 PM.

- Course overview:

  This is an extension of Calc I.

  We will use Calc I as a blueprint for this course.

  ↓ single-variable functions

  ↓ multivariable functions

  { limits, derivatives, integrals

  Applications: find volume, surface area, mass of an object,
  find min/max of a function,
  find circulation, flux of vector field, ...

* Students should try their best to attend lectures.

Calc I	Multivariable Calc
$f(x)$	$f(x,y,\dots)$: quantity that depends on several other quantities
graph is a curve	graph is a surface or solid
tangent line	tangent plane
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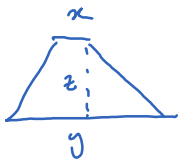
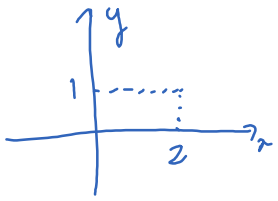
Ex $f(x,y) = xy$

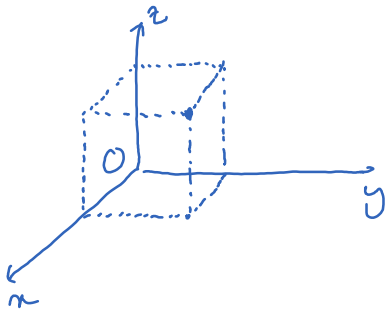
$f(x,y,z) = \frac{1}{2}(x+y)z$



The pair (x,y) represents a point in the Cartesian coordinate system:

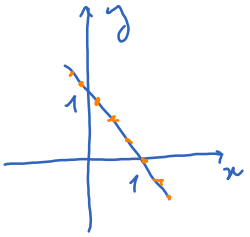


The triple (x,y,z) represents a point in the 3D Cartesian coordinate system:

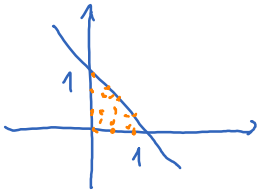


A set of points forms a shape. A shape can be described using equalities or inequalities.

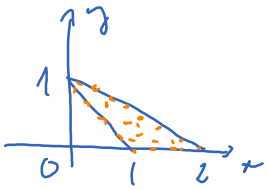
Ex:



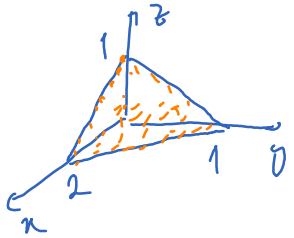
$\{(x,y) \mid x+y=1\}$



$\{(x,y) \mid x+y \le 1,\ x \ge 0,\ y \ge 0\}$

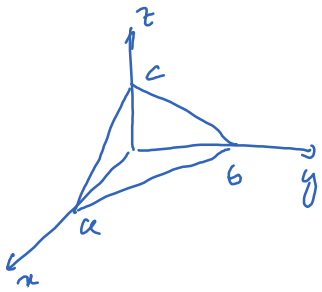


$\{(x,y) \mid x+y \ge 1,\ \frac{x}{2}+y \le 1,\ y \ge 0\}$



$\{(x,y,z) \mid \frac{x}{2}+y+z=1,\ x,y,z \ge 0\}$

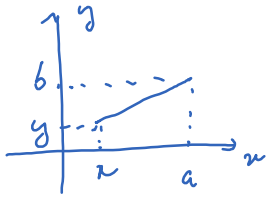
Note



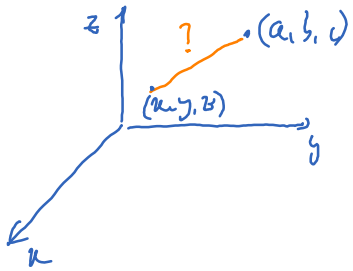
Eq. of plane with x, y, z intercepts as a, b, c:

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$$

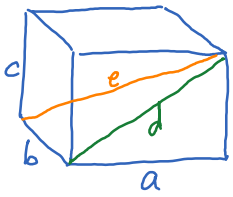
Distance



$$d = \sqrt{(x-a)^2 + (y-b)^2}$$



Think of Amazon box:

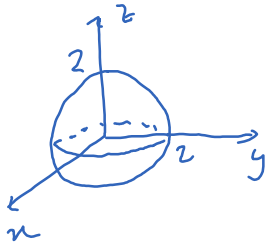


$$d = \sqrt{a^2 + c^2}$$

$$e = \sqrt{d^2 + b^2} = \sqrt{a^2 + b^2 + c^2}$$

Distance between (x, y, z) and (a, b, c) is $\sqrt{(x-a)^2 + (y-b)^2 + (z-c)^2}$.

Ex $\{(x, y, z) \mid x^2 + y^2 + z^2 = 4\}$ describes the sphere centered at the origin with radius 2.



More generally, the eq. of the sphere centered at (a, b, c) with radius r is

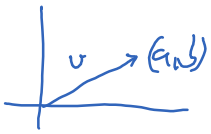
$$(x-a)^2 + (y-b)^2 + (z-c)^2 = r^2.$$

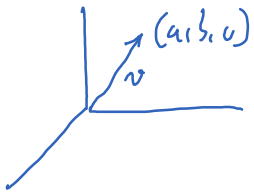
Vectors

Points are geometric objects. We can't perform calculations on them. Vectors are the version of points adapted for computing.

$v = (a, b)$

$v = (a, b, c)$



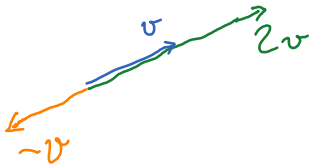


Addition

$$\left.\begin{aligned} v_1 &= (x, y) \\ v_2 &= (a, b) \end{aligned}\right\} \quad v_1 + v_2 = (x+a, y+b)$$

Geometrically, this is the parallelogram rule

Scaling



$v = (x, y, z)$

$cv = (cx, cy, cz)$